Information Matrix Fusion with Feedback Versus Number of Sensors

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Abstract **-** *Of late years, steady performance of two sensor information matrix fusion algorithm with different feedback strategy was evaluated by K.C.Chang, it was shown that, with non-full-rate communication, fusion performance of the algorithm with complete feedback is unstable, and increment of communication bandwidth just makes the performance worse as process noise is relatively large, while partial feedback strategy performs significantly better. As an extension and continue of above research, this paper pays more attentions to the effect of sensors number on the steady performance of the algorithm with different feedback strategy. Closed form analytical solution of steady state fused error covariance of the algorithm with arbitrary number of sensor is derived. Through simulation, a new phenomenon about algorithm performance is observed, it shows that for algorithm with complete feedback, as process noise is* ***relatively small rather large****, increase on the number of sensors does not improve but worsens the performance. Moreover, the algorithm with partial feedback performs much better than that with complete feedback. From another aspect, we verifies that quality of information is much more important than quantity.*

Keywords: Track Fusion; Multisensor data fusion; Multitarget Tracking; Hierarchical Fusion; Performance Evaluation.

1 Introduction

The problem of multitarget tracking utilizing information from multiple sensors has been received much attention over the last two decades[1-9]. Various solutions for this problem have been proposed, each with different trade-off between optimality and computation complexity. There are two basic architectures for fusion of multiple sensor information: *centralized* and *decentralized* (or *distributed*) (referred to as *measurement fusion* and *track fusion* in tracking area, respectively), depending on whether raw measurement data are sent to the fusion center or not. They have pros and cons in terms of optimality, communication bandwidth requirement, reliability, survivability, flexibility, etc. Centralized fusion is nothing but a conventional estimation problem with distributed data. Since the track estimate errors from different sensor are not necessarily independent, formulation optimal track fusion is more challenging and had been a focal point of fusion research for many years.

The pioneer work on the field of track fusion was developed under the assumption that the estimate errors of tracks to be fused are uncorrelated; the algorithm has been used extensively for its simple implementation. As local track estimate errors are correlated, this algorithm is over-confident and fusion error covariance obtained from the algorithm is not the actual error covariance. In [1], Bar-Shalom pointed out that, due to the same underlying process noise, local estimate errors are actually not independent, and an algorithm was derived to compute the error cross-covariance matrix between local estimates. Subsequently, an algorithm for fusion two tracks was derived which took the dependency of the errors into account by incorporating cross-covariance matrix into fusion formula [2]. Chang [3] pointed out that, due to lack prior information, the cross-covariance fusion algorithm is indeed only optimal in maximum likelihood sense. An optimal fusion algorithm based on information matrix approach was presented in [4], it focuses on decorrelation of common prior information and is not particularly effective in handling correlation brought by common process noise. The algorithm is optimal just when target model is deterministic (i.e. without process noise disturbance) or when real time communication between sensors and fusion center is employed. For many complicated sensors network, computing cross-covariance between local estimates is almost impossible, Julier and Uhlman proposed a covariance intersection fusion algorithm without assuming any knowledge on the correlation between local estimates to be fused [5]. As local estimates satisfy consistent condition, the algorithm yields consistent fusion estimates for any degree of cross correlation. However, the algorithm is sub-optimal compared with algorithm that exploits the correlation in estimates. X.R.Li [6] argued that most of the existing optimal fusion algorithms were obtained by ingenious but *ad hoc* manipulations of local estimates, and it appears that a systemic approach to develop distributed track fusion rules is still lacking. He developed a unified model for estimation fusion based upon the best linear unbiased estimation (BLUE) or linear minimum variance approach.

Recently, actual steady performance of two sensor information matrix fusion algorithm with different feedback strategy was evaluated by K.C.Chang [7,8] , it was shown that, with non-full-rate communication, fusion performance of algorithm with complete feedback degrades repaid as process noise is **relatively large**, and an interesting phenomenon is that, in complete feedback case, the more frequent the communication the more the fused covariance deviated from the true one. Partial feedback strategy performs significantly better. Note that above results are derived for two sensor only, and so there is a problem, it is that, if communication bandwidth is fixed, how does steady fusion performance behave with different feedback strategy as number of sensor varies ? Is it also the more sensors the worse the fusion performance in the complete feedback case? These problems are investigated in this paper. Closed form analytical solution of steady fused covariance of information matrix fusion with arbitrary number of sensor is derived. In the research, all sensors are assumed to be synchronized and no misassociation or merged measurement is considered.

The remainder of the paper is organized as follows. General distributed track fusion problem and information matrix fusion algorithm are reviewed in section 2. Then steady state fused covariance of information matrix fusion with arbitrary number of sensor is derived under the assumption of complete and partial feedback, respectively. According to results of section 3. Simulation and corresponding analysis is conducted in section 4.

2 Information matrix fusion

2.1 Track fusion problem statement

Consider a distributed tracking system in which $N(N \geq 2)$ sensors are tracking the same target. The mathematical model describing target dynamic is assumed to be linear time invariant and of the form

$$\mathbf{X}_{k+1} = \mathbf{F}\mathbf{X}_k + \mathbf{G}\mathbf{v}_k \qquad k = 0,1,2,\ldots \tag{1}$$

where, $\mathbf{X}_k \in \Re^{n_1}$ is state vector of target at time k and $\mathbf{F}$ is state transition matrix, $\mathbf{v}_k \in \Re^{n_2}$ is zero mean white Gaussian process noise with known covariance $\mathbf{Q}$, and $\mathbf{G}$ is the input matrix. The target is tracked by N sensors, where measurement model of sensor $j = 1,\ldots,N$ is described by

$$\mathbf{z}_k^{(j)} = \mathbf{H}^{(j)}\mathbf{X}_k + \mathbf{w}_k^{(j)} \tag{2}$$

where $\mathbf{w}_k^{(j)} \in \Re^{n_3}$ is zero-mean white Gaussian measurement noise with covariance $\mathbf{R}_k^{(j)}$.

It is assumed that local track estimates, $\hat{\mathbf{X}}_{k|k}^{(j)}$ and $\mathbf{P}_{k|k}^{(j)}$ $j = 1,\ldots,N$, are obtained by each sensor's Kalman filter based on measurement sequence $\mathbf{Z}_k^j = \{\mathbf{z}_i^j, i = 1,2,\ldots,k\}$ and are optimal in the sense of minimum variance. At the end of each n sampling interval, each sensor transmits its local estimate to fusion center where track association and fusion are performed. For fused estimate, there are two choice: either be sent back to sensor to improve local estimation performance or stored on fusion center. For the sake of simplicity, the dimension of the fused track and all local tracks are assumed to be the same. The distributed track fusion problem is to generate an "optimal" estimate $\hat{\mathbf{X}}_{k|k}$ from all local track information, i.e. $\hat{\mathbf{X}}_{k|k}^{(j)}$ and $\mathbf{P}_{k|k}^{(j)}$, and prior information about local and fused estimation if possible [9].

2.2 Information matrix fusion algorithm

The key idea of the information matrix fusion is identifying the common information shared by estimates that are to be fused, and then removing the information or de-correlation is implemented as fusion performs. It takes into account common information caused by prior information but not common process noise. The information matrix algorithm obtains fused track estimation at the fusion center as follows:

$$\mathbf{P}_{k|k}^{-1}\hat{\mathbf{X}}_{k|k} = \mathbf{P}_{k|k-n}^{-1}\hat{\mathbf{X}}_{k|k-n} + \sum_{j=1}^{N}\left\{\mathbf{P}_{k|k}^{(j)-1}\hat{\mathbf{X}}_{k|k}^{(j)} - \mathbf{P}_{k|k-n}^{(j)\ -1}\hat{\mathbf{X}}_{k|k-n}^{(j)}\right\} \tag{3}$$

$$\mathbf{P}_{k|k}^{-1} = \mathbf{P}_{k|k-n}^{-1} + \sum_{j=1}^{N}\left\{\mathbf{P}_{k|k}^{(j)-1} - \mathbf{P}_{k|k-n}^{(j)\ -1}\right\} \tag{4}$$

where the n step fusion state prediction and associated covariance are

$$\hat{\mathbf{X}}_{k|k-n} = \mathbf{F}^n\hat{\mathbf{X}}_{k-n|k-n} \tag{5}$$

$$\mathbf{P}_{k|k-n} = \mathbf{F}^n\mathbf{P}_{k-n|k-n}\mathbf{F}'^n + \sum_{i=1}^{n}\mathbf{F}^{n-i}\mathbf{G}\mathbf{Q}\mathbf{G}'\mathbf{F}'^{n-i} \tag{6}$$

and $\hat{\mathbf{X}}_{k|k-n}^{(j)}, \mathbf{P}_{k|k-n}^{(j)}$ are state prediction and corresponding covariance of local track j , respectively

As communication bandwidth is large enough, it is often hoped that local estimation performance can be improved through utilizing fusion information. The usual approach is to send back the latest fusion results to local sensors. Two feedback strategies are considered in [7,8], complete feedback and partial feedback. While former requires both the fused state estimate and associated covariance to be fed back and replace local ones, only the fused state estimate participates in feedback loop of latter. The following two equations hold in the first case, $\forall j = 1,\ldots,N$

$$\hat{\mathbf{X}}^{(j)}_{k|k-n} = \hat{\mathbf{X}}_{k|k-n} \tag{7}$$

$$\mathbf{P}^{(j)}_{k|k-n} = \mathbf{P}_{k|k-n} \tag{8}$$

and only (7) is valid in the second case.

3 Performance analysis of multisensor information matrix fusion with feedback

In this section, closed form analytical solution of steady fused covariance of information matrix fusion algorithm with $N(N \geq 2)$ sensors is derived under above the two feedback mechanism.

3.1 Complete feedback

In [7,8], steady state fused covariance of complete case with two sensors $(N = 2)$ is presented, however the result obtained there can not directly extend to the situation with $N > 2$ sensors, and new derivation is given below.

From (1) and (2), it is easy to show that the following two equations hold,

$$\mathbf{X}_k = \mathbf{F}^n \mathbf{X}_{k-n} + \sum_{i=1}^{n} \mathbf{F}^{n-i} \mathbf{G} \mathbf{v}_{k-n+i} \tag{9}$$

$$\mathbf{z}^{(j)}_{k-n+i} = \mathbf{H}^{(j)} \mathbf{F}^i \mathbf{X}_{k-n} + \mathbf{w}^{(j)}_{k-n+i} + \sum_{h=1}^{i} \mathbf{H}^{(j)} \mathbf{F}^{i-h} \mathbf{G} \mathbf{v}_{k-n+h} \tag{10}$$

For each local sensor $j = 1,\ldots,N$, it is possible to write,

$$\hat{\mathbf{X}}^{(j)}_{k|k} = \mathbf{P}^{(j)}_{k|k} \mathbf{P}^{(j)^{-1}}_{k|k-1} \mathbf{F} \hat{\mathbf{X}}^{(j)}_{k-1|k-1} + \mathbf{P}^{(j)}_{k|k} \mathbf{H}^{(j)'} \mathbf{R}^{(j)^{-1}} \mathbf{z}^{(j)}_k \tag{11}$$

Utilizing multiple iteration of (11) and (7), we have

$$\hat{\mathbf{X}}^{(j)}_{k|k} = \mathbf{A}^{(j)}_n \hat{\mathbf{X}}_{k-n|k-n} + \sum_{i=1}^{n} \mathbf{B}^{(j)}_i \mathbf{z}^{(j)}_{k-n+i} \tag{12}$$

where, $\forall i = 1,\ldots,n$, we have

$$\mathbf{A}^{(j)}_0 = \mathbf{I}, \qquad \mathbf{A}^{(j)}_i = \mathbf{A}^{(j)}_{i-1} \mathbf{P}^{(j)}_{k-i+1|k-i+1} \mathbf{P}^{(j)^{-1}}_{k-i+1|k-i} \mathbf{F}$$

$$\mathbf{B}^{(j)}_i = \mathbf{A}^{(j)}_{i-1} \mathbf{P}^{(j)}_{k-i+1|k-i+1} \mathbf{H}^{(j)'} \mathbf{R}^{(j)^{-1}} \tag{13}$$

Under the assumption of complete feedback, (3) and (4) can be rewritten as

$$\mathbf{P}^{-1}_{k|k} \hat{\mathbf{X}}_{k|k} = -(N-1) \mathbf{P}^{-1}_{k|k-n} \hat{\mathbf{X}}_{k|k-n} + \sum_{j=1}^{N} \mathbf{P}^{(j)^{-1}}_{k|k} \hat{\mathbf{X}}^{(j)}_{k|k} \tag{14}$$

$$\mathbf{P}^{-1}_{k|k} = -(N-1) \mathbf{P}^{-1}_{k|k-n} + \sum_{j=1}^{N} \mathbf{P}^{(j)^{-1}}_{k|k} \tag{15}$$

To compute the steady state error covariance of fused state estimate, subtracting $\mathbf{P}^{-1}_{k|k} \mathbf{X}_k$ from both sides of (14) and substituting (12) yields

$$\begin{aligned}
&\mathbf{P}^{-1}_{k|k} \left(\hat{\mathbf{X}}_{k|k} - \mathbf{X}_k \right) \\
&= -\mathbf{P}^{-1}_{k|k} \mathbf{X}_k - (N-1) \mathbf{P}^{-1}_{k|k-n} \hat{\mathbf{X}}_{k|k-n} + \sum_{j=1}^{N} \mathbf{P}^{(j)^{-1}}_{k|k} \hat{\mathbf{X}}^{(j)}_{k|k} \\
&= -(N-1) \mathbf{P}^{-1}_{k|k-n} \mathbf{F}^n \left(\hat{\mathbf{X}}_{k-n|k-n} - \mathbf{X}_{k-n} \right) - \mathbf{P}^{-1}_{k|k} \mathbf{X}_k \\
&\quad - (N-1) \mathbf{P}^{-1}_{k|k-n} \mathbf{F}^n \mathbf{X}_{k-n} \\
&\quad + \sum_{j=1}^{N} \mathbf{P}^{(j)^{-1}}_{k|k} \left[\mathbf{A}^{(j)}_n \hat{\mathbf{X}}_{k-n|k-n} + \sum_{i=1}^{n} \mathbf{B}^{(j)}_i \mathbf{z}^{(j)}_{k-n+i} \right]
\end{aligned} \tag{16}$$

Through simple algebra manipulation and substituting (10) , we can rewrite (16) as

$$\begin{aligned}
&\mathbf{P}^{-1}_{k|k} \left(\hat{\mathbf{X}}_{k|k} - \mathbf{X}_k \right) \\
&= \left\{ -(N-1) \mathbf{P}^{-1}_{k|k-n} \mathbf{F}^n + \sum_{j=1}^{N} \mathbf{P}^{(j)^{-1}}_{k|k} \mathbf{A}^{(j)}_n \right\} \\
&\quad \cdot \left(\hat{\mathbf{X}}_{k-n|k-n} - \mathbf{X}_{k-n} \right) + \sum_{j=1}^{N} \mathbf{P}^{(j)^{-1}}_{k|k} \mathbf{A}^{(j)}_n \mathbf{X}_{k-n} - \mathbf{P}^{-1}_{k|k} \mathbf{X}_k \\
&\quad - (N-1) \mathbf{P}^{-1}_{k|k-n} \mathbf{F}^n \mathbf{X}_{k-n} + \sum_{j=1}^{N} \left\{ \mathbf{P}^{(j)^{-1}}_{k|k} \sum_{i=1}^{n} \mathbf{B}^{(j)}_i \mathbf{z}^{(j)}_{k-n+i} \right\}
\end{aligned}$$

$$= \left\{ -(N-1)\mathbf{P}_{k|k-n}^{-1}\mathbf{F}^n + \sum_{j=1}^{N} \mathbf{P}_{k|k}^{(j)-1}\mathbf{A}_n^{(j)} \right\}$$

$$\cdot\left(\hat{\mathbf{X}}_{k-n|k-n} - \mathbf{X}_{k-n}\right) + \sum_{j=1}^{N} \mathbf{P}_{k|k}^{(j)-1}\mathbf{A}_n^{(j)}\mathbf{X}_{k-n}$$

$$-(N-1)\mathbf{P}_{k|k-n}^{-1}\mathbf{F}^n\mathbf{X}_{k-n} + \sum_{j=1}^{N}\left\{\mathbf{P}_{k|k}^{(j)-1}\sum_{i=1}^{n}\mathbf{B}_i^{(j)}\mathbf{w}_{k-n+i}^{(j)}\right\}$$

$$-\mathbf{P}_{k|k}^{-1}\mathbf{X}_k + \sum_{j=1}^{N}\mathbf{P}_{k|k}^{(j)-1}\sum_{i=1}^{n}\mathbf{B}_i^{(j)}\mathbf{H}^{(j)}\mathbf{F}^i\mathbf{X}_{k=n}$$

$$\sum_{j=1}^{N}\mathbf{P}_{k|k}^{(j)-1}\sum_{i=1}^{n}\mathbf{B}_i^{(j)}\sum_{h=1}^{i}\mathbf{H}^{(j)}\mathbf{F}^{i-h}\mathbf{G}\mathbf{v}_{k-n+h} \tag{17}$$

It has been proven in [7] that $\mathbf{A}_n^{(j)}$ satisfies the following identity

$$\mathbf{A}_n^{(j)} = -\sum_{i=1}^{n}\mathbf{B}_i^{(j)}\mathbf{H}^{(j)}\mathbf{F}^i + \mathbf{F}^n \tag{18}$$

Substituting (18) and (15) into (17), we have

$$\mathbf{P}_{k|k}^{-1}\left(\hat{\mathbf{X}}_{k|k} - \mathbf{X}_k\right)$$

$$= \left\{ -(N-1)\mathbf{P}_{k|k-n}^{-1}\mathbf{F}^n + \sum_{j=1}^{N} \mathbf{P}_{k|k}^{(j)-1}\mathbf{A}_n^{(j)} \right\}$$

$$\cdot\left(\hat{\mathbf{X}}_{k-n|k-n} - \mathbf{X}_{k-n}\right) + \sum_{j=1}^{N} \mathbf{P}_{k|k}^{(j)-1}\mathbf{A}_n^{(j)}\mathbf{X}_{k-n}$$

$$-(N-1)\mathbf{P}_{k|k-n}^{-1}\mathbf{F}^n\mathbf{X}_{k-n} + \sum_{j=1}^{N}\mathbf{P}_{k|k}^{(j)-1}\sum_{i=1}^{n}\mathbf{B}_i^{(j)}w_{k-n+i}^{(j)}$$

$$-\mathbf{P}_{k|k}^{-1}\mathbf{X}_k + \sum_{j=1}^{N}\mathbf{P}_{k|k}^{(j)-1}\left(\mathbf{F}^n - \mathbf{A}_n^{(j)}\right)\mathbf{X}_{k-n}$$

$$\sum_{j=1}^{N}\mathbf{P}_{k|k}^{(j)-1}\sum_{i=1}^{n}\mathbf{B}_i^{(j)}\sum_{h=1}^{i}\mathbf{H}^{(j)}\mathbf{F}^{i-h}\mathbf{G}v_{k-n+h}$$

$$= \left\{ -(N-1)\mathbf{P}_{k|k-n}^{-1}\mathbf{F}^n + \sum_{j=1}^{N} \mathbf{P}_{k|k}^{(j)-1}\mathbf{A}_n^{(j)} \right\}$$

$$\cdot\left(\hat{\mathbf{X}}_{k-n|k-n} - \mathbf{X}_{k-n}\right) + \sum_{j=1}^{N}\left\{\mathbf{P}_{k|k}^{(j)-1}\sum_{i=1}^{n}\mathbf{B}_i^{(j)}w_{k-n+i}^{(j)}\right\}$$

$$\sum_{i=1}^{n}\left\{\sum_{j=1}^{N}\left(\mathbf{P}_{k|k}^{(j)-1}\sum_{h=i}^{n}\mathbf{B}_h^{(j)}\mathbf{H}^{(j)}\mathbf{F}^{h-i}\right) - \mathbf{P}_{k|k}^{-1}\mathbf{F}^{n-i}\right\}\cdot\mathbf{G}v_{k-n+i} \tag{19}$$

Define the steady state MMSE matrix of the fused estimate $\hat{\mathbf{X}}_{k|k}$ as

$$\mathbf{\Omega}_x = \lim_{k\to\infty}\mathbf{\Omega}_x(k) = \lim_{k\to\infty}E\left[\left(\hat{\mathbf{X}}_{k|k} - \mathbf{X}_k\right)\left(\hat{\mathbf{X}}_{k|k} - \mathbf{X}_k\right)'\right]$$

From (19), It is easy to show that $\mathbf{\Omega}_x$ satisfies the following discrete Lyapunov equation,

$$\mathbf{\Omega}_x = \mathbf{C}_f\mathbf{\Omega}_x\mathbf{C}_f' + \mathbf{\Omega}_f \tag{20}$$

where

$$\mathbf{C}_f = \lim_{k\to\infty}\mathbf{P}_{k|k}\left\{-(N-1)\mathbf{P}_{k|k-n}^{-1}\mathbf{F}^n + \sum_{j=1}^{N}\mathbf{P}_{k|k}^{(j)-1}\mathbf{A}_n^{(j)}\right\}$$

$$\mathbf{\Omega}_f = \sum_{j=1}^{N}\sum_{i=1}^{n}\mathbf{W}_s^{(j)}(i)\mathbf{R}^{(j)}\mathbf{W}_s^{(j)}(i)' + \sum_{i=1}^{n}\mathbf{V}_s(i)\mathbf{G}\mathbf{Q}\mathbf{G}'\mathbf{V}_s(i)'$$

$$\mathbf{W}_s^{(j)}(i) = \lim_{k\to\infty}\mathbf{P}_{k|k}\mathbf{P}_{k|k}^{(j)-1}\mathbf{B}_i^{(j)}$$

$$\mathbf{V}_s^{(j)}(i)$$

$$= \lim_{k\to\infty}\mathbf{P}_{k|k}\left\{\sum_{j=1}^{N}\left(\mathbf{P}_{k|k}^{(j)-1}\sum_{h=i}^{n}\mathbf{B}_h^{(j)}\mathbf{H}^{(j)}\mathbf{F}^{h-i}\right) - \mathbf{P}_{k|k}^{-1}\mathbf{F}^{n-i}\right\} \tag{21}$$

3.2 Partial feedback

In the case of partial feedback, only (7) is valid and (3) and (4) can be formulated as follows.

$$\mathbf{P}_{k|k}^{-1}\hat{\mathbf{X}}_{k|k} = \mathbf{P}_{k|k-n}^{-1}\hat{\mathbf{X}}_{k|k-n} + \sum_{j=1}^{N}\left\{\mathbf{P}_{k|k}^{(j)-1}\hat{\mathbf{X}}_{k|k}^{(j)} - \mathbf{P}_{k|k-n}^{(j)\ -1}\hat{\mathbf{X}}_{k|k-n}\right\} \tag{22}$$

$$\mathbf{P}_{k|k}^{-1} = \mathbf{P}_{k|k-n}^{-1} + \sum_{j=1}^{N}\left\{\mathbf{P}_{k|k}^{(j)-1} - \mathbf{P}_{k|k-n}^{(j)\ -1}\right\} \tag{23}$$

Note that changing the value of N does not alter the forms of (22) and (23) and only length of summation item need to be adjusted. Therefore, steady fused covariance of two sensors information matrix fusion derived in [7] can be generalized to situation of $N(N>2)$ sensors. Like the case of complete feedback, there is also a discrete Lyapunov equation,

$$\mathbf{\Omega}_x = \mathbf{C}_p\mathbf{\Omega}_x\mathbf{C}_p' + \mathbf{\Omega}_p \tag{24}$$

where

$$\mathbf{C}_p = \lim_{k\to\infty} \mathbf{P}_{k|k} \left[\sum_{j=1}^{N} \left(\mathbf{P}_{k|k}^{(j)-1} \mathbf{A}_n^{(j)} - \mathbf{P}_{k|k-n}^{(j)\ -1} \mathbf{F}^n \right) + \mathbf{P}_{k|k-n}^{-1} \mathbf{F}^n \right] \tag{25}$$

with Ω_p has the same definition of Ω_f in (21).

Therefore, the steady state fusion error covariance matrix Ω_x in both complete and partial feedback cases can be computed analytically by solving (20) and (24), this can be easily achieved by *dlyap* function of MATLAB.

4 Simulation and Analysis

In order to demonstrate the relationship between actual steady state fusion error covariance of the algorithm with feedback and the number of sensors, simulation is done in this section. The target model is

$$\mathbf{X}_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \mathbf{X}_k + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \mathbf{v}_k \tag{26}$$

with sampling interval $T = 1$ and $\mathbf{v}_k$ is zero-mean white Gaussian process noise with variance q . The measurements of sensor j are modeled as

$$\mathbf{z}_k^{(j)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{X}_k + \mathbf{w}_k^{(j)} \qquad j = 1,\ldots,N \tag{27}$$

where $\mathbf{w}_k^{(j)}$ is zero-mean white Gaussian sequence with variance $R^{(j)} = 1$, measurement noise of different sensor are independent identical distribution (IID). Local tracks are transmitted to fusion center at the end of each two sampling intervals, i.e. $n = 2$ and the value of N is set as 2, 3, 4, respectively

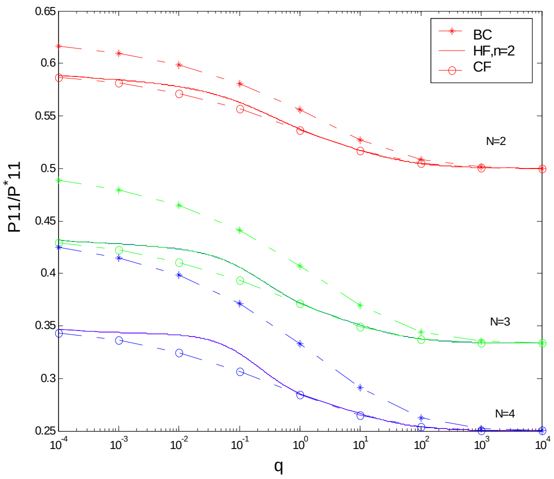

Fig.1. Ratio of P11 for information matrix fusion with completel feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

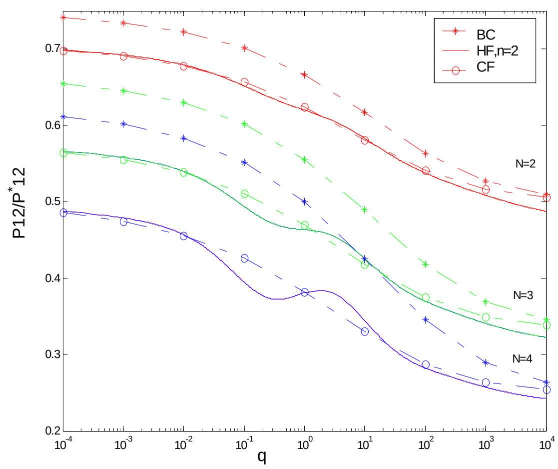

Fig.2. Ratio of P12 for information matrix fusion with completel feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

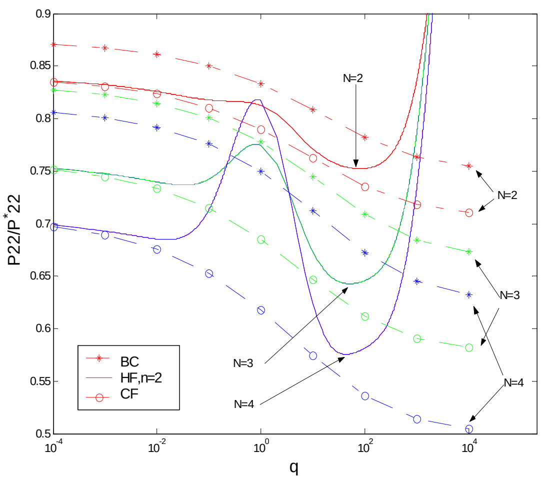

Fig.3. Ratio of P22 for information matrix fusion with completel feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

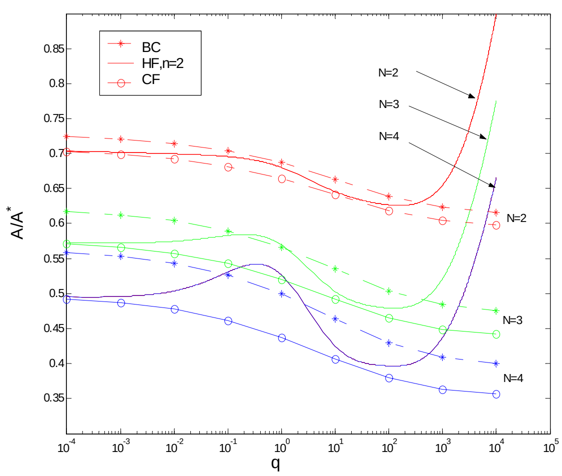

Fig.4. Ratio of error ellipses aera for information matrix fusion with completel feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

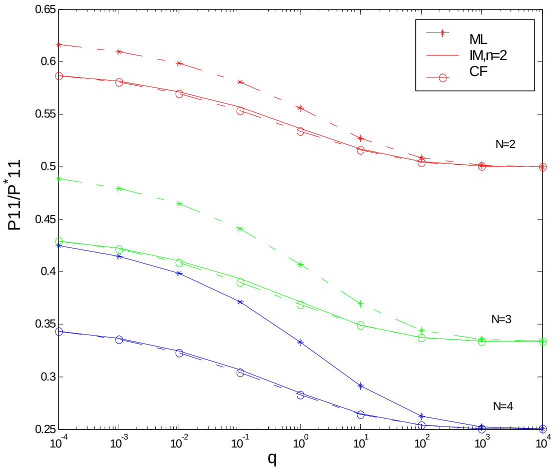

Fig.5. Ratio of P11 for information matrix fusion with partial feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

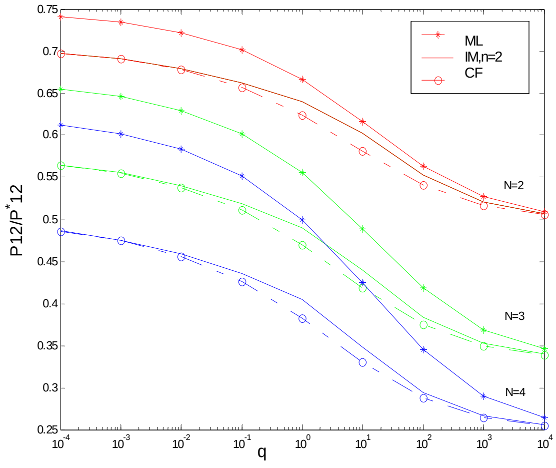

Fig.6. Ratio of P12 for information matrix fusion with partial feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

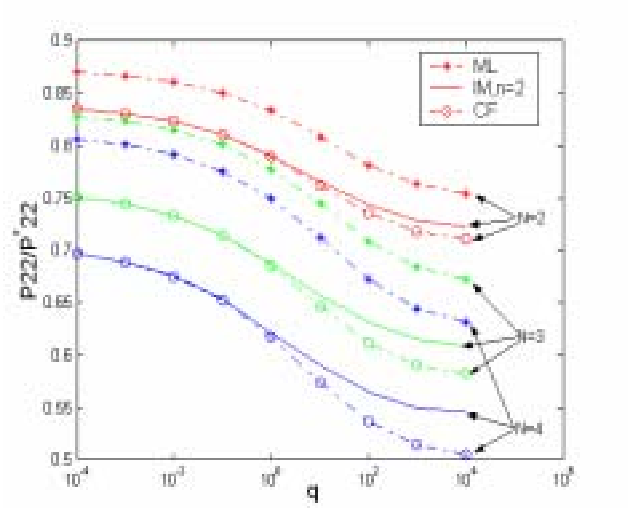

Fig.7. Ratio of P22 for information matrix fusion with partial feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

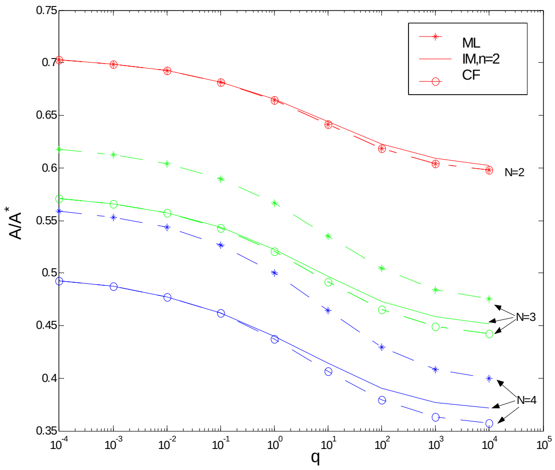

Fig.8. Ratio of error ellipses aera for information matrix fusion with partial feedback and half data rate versus that of single sensor optimal estimation with different number of sensors

Figs.1-3 show the theoretical results algorithm with complete feedback. The ratios of elements of steady state error covariance matrix Ω_c of information matrix fusion with different N and steady state error covariance matrix Ω^* of single sensor optimal estimation over a wide range of process noise q are plotted. The ratios of areas of error ellipses, A/A^*, are given in Fig.4. The dash-dot lines with asterisk marker and circle marker, respectively, are the theoretical results based on cross covariance algorithm (only optimal in maximum likelihood sense, labeled as BC in Figs.1-4 and ML in Figs.5-8, respectively) and optimal centralized fusion (equivalent to information matrix fusion with $n=1$). The solid lines are the theoretical results based on (20).

As can be seen from these figures, performance of information matrix fusion with half rate communication and complete feedback appears to be unstable. Fig.1-2 indicate that there exist fluctuation for error covariance of position and position-velocity as $q \le 10$, and the more sensors the more intensively the performance fluctuates. As $10^2 \le q \le 10^4$, they restore to approach that of cross covariance fusion again. Compared with situations of position and position-velocity, the fluctuation phenomenon in velocity estimation (fig.3) is more obvious as $10^{-2} \le q \le 10$ and performance degradation over the range approaches a peak as q is about 1. Notice that, in particular point of the noise range, performance of 4 sensors system is even not good as that of 2 sensors system, increasing number of sensors makes the performance worse when process noise is relatively small. Moreover, as $q \geq 10^2$, the performance of velocity estimation deteriorates rapidly and estimation errors diverge, fusion can not bring any benefit at this moment. Figure 4 shows overall performance of information matrix fusion with complete feedback and half rate communication.

From Figs.1-4, two conclusions can be obtained about complete feedback case. Firstly, whichever way you do it, increase communication rate or number of sensor, in specified process noise range, both make performance worse. Secondly, compared with performance of the system with fixed sensor number and changeable communication rate, performance degradation of the system with fixed communication rate and changeable sensor number happens more “early” in terms of process noise q.

Figs.5-8 show the theoretical results of algorithm with half date rate and partial feedback. It can be seen from these figures that performance of the case is much better than that of complete case, in particular performance instability of complete feedback case is eliminated here. As process noise is relatively small ($10^{-4} \leq q \leq 1$), the performance difference of the algorithm from the optimal centralized fusion algorithm is almost indistinguishable. As process noise is relatively large ($10 \leq q \leq 10^4$), although the algorithm performance degrades a little and moreover the more sensors the more apparent performance degrades, it is still better than that of corresponding cross covariance algorithm. This is not surprising because only unbiased fused state estimate $\mathbf{X}_{k|k}$ is sent back to local sensors, which still use their own error covariance matrix, hence affection of poor quality fused error covariance is minimized to lowest level.

5 Conclusions

As the continuator and extension of [7,8], steady state performances of information matrix fusion algorithm with different feedback strategy are investigated in this paper and focal point is relationship of performance and the number of sensors. Closed form analytical solution of actual steady state fused error covariance matrix with arbitrary number of sensor is presented.

We show that, in the complete feedback case, performance deterioration of the system with fixed communication rate and changeable sensor number happens more “early”, in terms of process noise q, than that of system with fixed sensor number and changeable communication rate.

We also show that performance of the algorithm with partial feedback is much better than that of complete feedback algorithm again.

Theses results verify existed conclusion that, from other aspect, quality of information to be fused is much more important than quantity.

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